Practical measures of integrated information for stationary systems

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Integrated information

Integrated information, $\Phi$, quantifies the extent to which a system as a whole generates more information than the sum of its parts. $\Phi$ has been suggested to measure the quantity of consciousness generated by a system [1,2], and shares with similar approaches [3] an emphasis on measuring conjoint integration and segregation in a system’s dynamics, reflecting basic properties of conscious phenomenology.

$\Phi_{\text{MIB}}$

The latest formulation, $\Phi_{\text{MIB}}$, measures the integrated information generated when a system transitions into a particular state [1,2]. Information is quantified as the reduction in entropy of a hypothetical past state that, at a priori, had maximum entropy.

$\Phi_{\text{Stat}}$

Measuring $\Phi_{\text{MIB}}$ requires 2 assumptions that deny application to real neural systems. Namely, that the system be

1. **Memoryless**: The maximum entropy distribution can only be imposed as an initial condition. In the brain we only observe states with history. Hypothetical evolution of a state without history can only be characterized if the system has no memory.
2. **Discrete**: Continuous variables don’t have a unique maximum entropy distribution.

Stationary $\Phi$ ($\Phi_{\text{Stat}}$): definition

We propose a new measure, $\Phi_{\text{Stat}}$, based on information generated by the current state about an actual past state. This enables application to any stationary time-series data.

- The effective information (EI) with respect to bipartition $B = \{M^1, M^2\}$ is the information generated by the whole $X$ minus the sum of that of the parts. For current state $x = (m^1, m^2)$, the EI for timescale $\tau$ is

$$\text{EI}(x, \tau, B) = I(X_{-\tau} |X_\tau = x) - \sum_{i=1}^2 I(M^i_{-\tau} |M^i_\tau = m^i).$$  

- The minimum information bipartition (MIB) minimizes normalized EI:

$$B_{\text{MIB}}(x, \tau) = \arg \min_B \left\{ \frac{\text{EI}(x, \tau, B)}{\log |B|} \right\}.$$  

$$K_{\text{MIB}}(\{M^1, M^2\}) = \min_B \|H(M^i_\tau)\|.$$  

Normalization is necessary because sub-systems that are almost as large as the whole generate almost as much information as the whole; we require a bias towards approximately even bipartitions.

- Finally, the integrated information is the EI with respect to the MIB:

$$\Phi_{\text{Stat}}(x, \tau) = \text{EI}(x, \tau, B_{\text{MIB}}(x, \tau)).$$

$\Phi_{\text{Stat}}$ in practice

$\Phi_{\text{Stat}}$ can be computed numerically from time-series and analytically given a generative model. For Gaussian systems, only covariance matrices are required [4], making it extremely easy to apply in practice.

- $\Phi_{\text{Stat}}$ computed analytically and from simulated time-series for example networks. Simulations typically give accurate results, but network (g) exhibits instability.

Optimized networks with (h) 2 equal afferents per node, $\Phi_{\text{MIB}} = 0.245$; (i) all afferents to a given node equal, $\Phi_{\text{Stat}} = 0.30$. (Constant total afferent.)

Auto-regressive $\Phi$ (AR$\Phi$)

For non-Gaussian systems we introduce an alternative measure, AR$\Phi$, based on how well the present predicts the past, but only to the extent that predictions based on the whole outstrip predictions based on parts independently. We consider the regression

$$X_{i-\tau} = A \cdot X_i + \epsilon_i,$$

and define predictive power (PP) as

$$\text{PP}(X_{i-\tau} | X_i) = \log |\text{COV}(X_{i-\tau})| - \log |\text{COV}(\epsilon_i)|.$$  

(For a part $M$, substitute $M$ for $X$.) Then construct AR$\Phi$ analogously to $\Phi_{\text{Stat}}$, replacing $f$ with PP.

- AR$\Phi$ is equivalent to $\Phi_{\text{Stat}}$ for Gaussian systems [4], whereas for non-Gaussian systems it provides a pragmatic alternative to $\Phi_{\text{Stat}}$.

- AR$\Phi$ uses linear regression of the present to predict the past. It thus invites comparison with causal density [5], which uses regression of the present to predict the future.

Instability

Varying connection 6 $\rightarrow$ 1 in network (h) results in a discontinuity in $\Phi_{\text{Stat}}$. This arises from using normalized EI to find the MIB but non-normalized EI to determine $\Phi$.

Conclusion

- Our measures provide new opportunities for empirically testing the relationship between integrated information and consciousness.

- AR$\Phi$ (and $\Phi_{\text{Stat}}$ for Gaussian systems) are state-independent. If they correlate with consciousness, then conscious level (i) is constant during each stationary epoch in brain activity: (ii) changes when functional connectivity changes.

- $\Phi_{\text{Stat}}$ and AR$\Phi$ use the stationary distribution and measure a ‘process’. By contrast, $\Phi_{\text{MIB}}$ uses the maximum entropy distribution and measures a ‘capacity’.

- The normalization-instability challenges acquisition of physical meaning to $\Phi$.